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EFFECTS OF TEMPERATURE DEPENDENT THERMAL CONDUCTIVITY ON MAGNETOHYDRODYNAMIC FREE CONVECTION FLOW ALONG A VERTICAL FLAT PLATE

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ABSTRACT
The effects of temperature dependent thermal conductivity on magnetohydrodynamic (MHD) free convection flow of an electrically conducting fluid along a vertical flat plate have been investigated here. The governing equations with associated boundary conditions for this phenomenon are converted to dimensionless forms using a suitable transformation. The transformed non-linear equations are then solved using the implicit finite difference method together with Keller-box scheme. Numerical results of the velocity and temperature profiles, surface temperature and rate of heat transfer for different values of the magnetic parameter, thermal conductivity variation parameter, Prandtl number, heat generation and joule heating parameter**s** are presented graphically. Detailed discussion is given for the effects of the aforementioned parameters.

Keywords: Thermal Conductivity Variation, Joule Heating, Rate Of Heat Transfer.

1. INTRODUCTION

Electrically conducting fluid flow in presence of magnetic field, the effect of temperature dependent conductivity on MHD flow with heat conduction problems are important from the technical point of view and such types of problems have received much attention by many researchers.

Magnetohydrodynamics is that branch of science, which deals with the motion of highly conducting ionized (electric conductor) fluid in presence of magnetic field. The motion of the conducting fluid across the magnetic field generates electric currents which change the magnetic field and the action of the magnetic field on these currents give rise to mechanical forces, which modify the fluid. It is possible to attain equilibrium in a conducting fluid if the current is parallel to the magnetic field. In the case when the conductor is either a liquid or a gas, electromagnetic forces will be generated which may be of the same order of magnitude as the hydrodynamical and inertial forces. Thus the equation of motion as well as the other forces will have to take these electromagnetic forces into account.

Convection is the transfer of heat energy in a gas or liquid by movement of currents. Considerable convection is responsible for making macaroni rise and fall in a pot of heated water. The warmer portions of the water are less dense and therefore, they rise and. the cooler portions of the water fall because they are denser.

Model studies of the free convection flows have earned reputations because of their applications in geophysical, geothermal and nuclear engineering problems. Akther [7] published journal articles.

The present study is to incorporate the idea of the effects of temperature dependent thermal conductivity on MHD free convection boundary layer flow along a vertical flat plat. The governing boundary layer equations are transformed into a non-dimensional form and the resulting non-linear system of partial differential equations is reduced to local non-similar partial differential forms by adopting appropriate transformations. The transformed boundary layer equations are then solved numerically. Numerical results of the velocity, temperature, surface temperature and heat transfer for the magnetic, thermal conductivity variation, Prandtl number, heat generation and joule heating parameter**s** are presented graphically.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

We consider a steady two dimensional laminar free convection flow of an electrically conducting, viscous, incompressible fluid along a vertical flat plate of length l, thickness b (Fig.-1). It is assumed that the temperature at the outer surface of the plate is maintained at a constant temperature T_b, where $T_b > T_\infty$, the ambient temperature of fluid. A uniform magnetic field strength H_0 is imposed along the $\bar{y}$ -axis i.e. normal direction to the surface and $\bar{x}$ -axis is taken along the plate. The coordinate system and the configuration are shown in Fig.-1.

So the governing equations under Boussinesq approximations for this present problem of continuity, momentum, energy equations take the form

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \quad (1)$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}=\nu\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}+g\beta(T_f-T_\infty)-\frac{\sigma\ H_0^2\ \bar{u}}{\rho} \qquad (2)$$

$$\bar{u}\frac{\partial T_f}{\partial \bar{x}}+\bar{v}\frac{\partial T_f}{\partial \bar{y}}=\frac{1}{\rho\ c_p}\frac{\partial}{\partial \bar{y}}(\kappa_f\frac{\partial T_f}{\partial \bar{y}})+\frac{Q_0}{\rho C_p}(T_f-T_\infty)+\sigma\frac{H_0^2\bar{u}^2}{\rho C_p} \qquad (3)$$

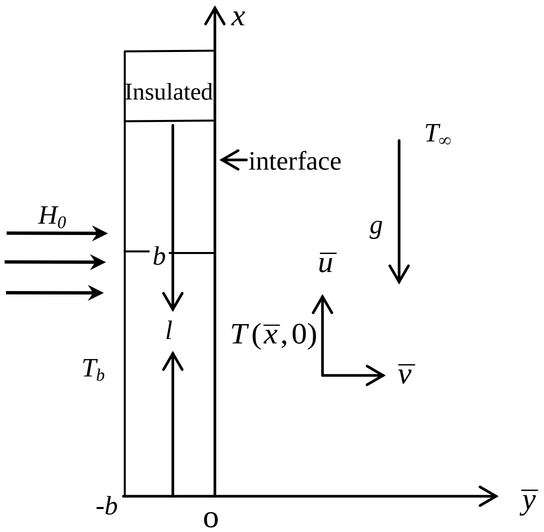

Fig 1. Physical model and coordinate system

Here. β is coefficient of volume expansion. The temperature dependent thermal conductivity, which is used by Rahaman [2] as follows

$$\kappa_f=\kappa_\infty[1+\delta(T_f-T_\infty)] \qquad (4)$$

Where κ_∞ is the thermal conductivity of the ambient fluid and δ is a constant, defined as $\delta=\frac{1}{\kappa_f}\left(\frac{\partial\kappa}{\partial T}\right)_f$. The appropriate boundary condition to be satisfied by the above equations are

$$\left.\begin{aligned}&\bar{u}=0,\ \bar{v}=0\\&T_f=T(\bar{x},0),\ \frac{\partial T_f}{\partial \bar{y}}=\frac{\kappa_s}{b\kappa_f}(T_f-T_b)\end{aligned}\right\}\ on\ \bar{y}=0,\ \bar{x}>0 \qquad (5)$$

$$\bar{u}\to 0,\ T_f\to T_\infty\quad as\ \ \bar{y}\to\infty,\ \bar{x}>0$$

The dimensionless governing equations and boundary conditions obtained by using these dimensionless quantities

$$x=\frac{\bar{x}}{l},\ \ y=\frac{\bar{y}}{l}Gr^{\frac{1}{4}},\ u=\frac{\bar{u}l}{\upsilon}Gr^{-\frac{1}{2}},\ v=\frac{\bar{v}l}{\upsilon}Gr^{-\frac{1}{4}}, \qquad (6)$$

$$\theta=\frac{T_f-T_\infty}{T_b-T_\infty},\ Gr=\frac{g\beta l^3(T_b-T_\infty)}{\upsilon^2}$$

where Gr is the Grashof number, θ is the dimensionless temperature.

From (1)-(3), we get the dimensionless equations

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (7)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+Mu=\frac{\partial^2 u}{\partial y^2}+\theta \qquad (8)$$

$$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{\Pr}(1+\gamma\theta)\frac{\partial^2\theta}{\partial y^2}+\frac{\gamma}{\Pr}\left(\frac{\partial\theta}{\partial y}\right)^2+Ju^2+Q\theta \qquad (9)$$

where $\Pr=\frac{\mu\ c_p}{\kappa_\infty}$ is Prandtl number, $M=\frac{\sigma\ H_0^{\ 2}l^2}{\mu\ Gr^{1/2}}$ is magnetic parameter, $\gamma=\delta(T_b-T_\infty)$ is thermal conductivity variation parameter, $J=\frac{\sigma H_0^2\nu Gr^{1/2}}{\rho C_p(T_b-T_\infty)}$ is joule heating parameter and $Q=\frac{Q_0 l^2}{\mu C_p Gr^{1/2}}$ is heat generation parameter. All are dimensionless parameters. The corresponding boundary conditions (5) then follows

$$u=0,\ v=0,\ \ \theta-1=(1+\gamma\theta)\ p\frac{\partial\theta}{\partial y}\quad on\ \ y=0,\ x>0 \qquad (10)$$

$$u\to 0,\ \theta\to 0\quad as\ \ y\to\infty,\ x>0$$

here $p=\left(\frac{\kappa_\infty\ b}{\kappa_s l}\right)Gr^{\frac{1}{4}}$ is the conjugate conduction parameter. In the present work, it is considered p = 1.

To solve the equations (8)-(9) by boundary conditions (10) the following transformations are proposed by Merkin & Pop [4]

$$\psi=x^{4/5}(1+x)^{-1/20}f(x,\eta)$$

$$\eta=yx^{-1/5}(1+x)^{-1/20} \qquad (11)$$

$$\theta=x^{1/5}(1+x)^{-1/5}h(x,\eta)$$

here η is the similarity variable and ψ is the non-dimensional stream function which satisfies the continuity equation and is related to the velocity components in the usual way as

$$u=\frac{\partial\psi}{\partial y}\ \text{and}\ v=-\frac{\partial\psi}{\partial x}.$$

Here, $h(x,\eta)$ represents the dimensionless temperature. The momentum and energy equations are transformed for the new coordinate system as

$$f'''+\frac{16+15x}{20(1+x)}ff''-\frac{6+5x}{10(1+x)}f'^2$$

$$-Mx^{\frac{2}{5}}(1+x)^{\frac{1}{10}}f'+h=x\ \left(f'\frac{\partial f'}{\partial x}-f''\frac{\partial f}{\partial x}\right) \qquad (12)$$

(6)

$$\frac{1}{\Pr}\ h''+\frac{\gamma}{\Pr}\left(\frac{x}{1+x}\right)^{\frac{1}{5}}hh''+\frac{\gamma}{\Pr}\left(\frac{x}{1+x}\right)^{\frac{1}{5}}h'^2$$

$$+\frac{16+15x}{20(1+x)}fh'-\frac{1}{5(1+x)}f'h=\left(f'\frac{\partial h}{\partial x}-h'\frac{\partial f}{\partial x}\right) \qquad (13)$$

where prime denotes partial differentiation with respect to η. The boundary conditions for (10) then take the form

$$f(x,0) = f'(x,0) = 0$$

$$h'(x,0) = \frac{x^{1/5}(1+x)^{-1/5}h(x,0) - 1}{(1+x)^{-1/4} + \gamma x^{1/5}(1+x)^{-9/20}h(x,0)} \quad (14)$$

$$f'(x,\infty) \to 0, h(x,\infty) \to 0$$

It is important to calculate the values of the surface temperature and it is obtained from this relation

$$\theta(x,0) = x^{\frac{1}{5}}(1+x)^{-\frac{1}{5}}h(x,0) \quad (15)$$

In practical point of view, it is important to calculate the values of the local rate of heat transfer from this non-dimensional form as

$$N_u = (Gr^{-\frac{1}{4}}/\kappa_f(T_b - T_\infty))q_w \quad (16)$$

where $q_w = -\kappa_f(\frac{\partial T_f}{\partial \bar{y}})_{\bar{y}=0}$ is the heat flux. Thus the local rate of heat transfer is

$$N_{ux} = -(1+x)^{-\frac{1}{4}}h'(x,0) \quad (17)$$

3. METHOD OF SOLUTION

In this paper investigated the effects of the temperature dependent thermal conductivity on electrically conducting fluid in free convection flow along a vertical flat plate with heat generation and joule heating for strong magnetic field. Along with the boundary conditions (14), the solution of the parabolic non-linear ordinary differential equations (12) and (13) will be found by using the implicit finite difference method together with Keller-box elimination technique [5] which is well documented by Cebeci and Bradshaw [6].

4. RESULT AND DISCUTION

In this simulation the values of the Prandtl number *Pr* are considered as 0.733, 1.099, 1.63 &2.18 that correspond to hydrogen, water, glycerin&sulfur dioxide respectively.

The velocity and the temperature profiles obtained from the solutions of equations (12) and (13) are depicted in figures 2 to 6. Also the surface temperature and the local rate of heat transfer profiles obtained from the solutions of equations (15) and (17) are depicted in figures 7 to 11.

In Fig. 2(a), it is shown that the magnetic field action retards the fluid velocity with γ = 0.01, *Pr* = 0.733, *Q* = 0.01 and *J* = 0.01. Here position of peak velocity moves toward the interface with the increasing *M*. From fig. 2(b), it can be observed that the temperature within the boundary layer increases for increasing values of *M*.

The effect of thermal conductivity variation parameter γ on velocity and temperature against η within boundary layer with *M* = 0.01, *Pr* = 0.733, *Q* = 0.01 and *J* = 0.01 are shown in fig. 3(a), 3(b) respectively. It is seen that the velocity and temperature increase within boundary layer with increasing γ. It means that the velocity and thermal boundary layer thickness increase for large values of γ.

Fig. 4(a) and 4(b) illustrate the velocity and temperature against η for different values of *Pr* with *M* = 0.01, γ = 0.01, *Q* = 0.01 and *J* = 0.01. From fig. 4(a), it can be observed that the velocity decreases as well as its position moves toward the interface with the increasing *Pr*. From fig. 4(b), it is seen that the temperature profiles shift downward with the increasing values of *Pr*.

Figures 5(a)-(b) describe the velocity and temperature against η for different values of heat generation parameter *Q* with *M* = 0.01, γ = 0.01, *Pr* = 0.733 and *J* = 0.01. From fig. 5(a), it can be observed that the velocity increases as well as its position moves toward the interface with the increasing *Q*. Fig. 5(b), shows the temperature also same as increasing within the boundary layer. It means that the velocity and thermal boundary layer thickness increase for large values of *Q*.

The effect of joule heating parameter *J* on the velocity and temperature against η within the boundary layer with *M* =0.01, γ = 0.01, *Pr* =0.733 and *Q* =0.01 are shown in fig. 6(a), 6(b) respectively. The velocity and temperature increase within boundary layer with increasing *J*.

The variation of the surface temperature $\theta(x,0)$ and local rate of heat transfer N_{ux} against *x* for different values of *M* with γ = 0.01, *Pr* = 0.733, *Q* = 0.01 and *J* = 0.01 at different positions are illustrated in fig. 7(a), 7(b) respectively. The velocity decreases as fig. 2(a) due to the increasing *M*. It is observed from fig. 7(a) that the increased value of magnetic parameter *M* leads to increase the surface temperature factor on the plate. But the temperature within the boundary layer increases (Fig. 2(b)) for increasing *M*. As a result, fig. 7(b) shows that the heat transfer rate from the plate to the fluid decreases due to the increased value of *M*. The magnetic field acts against the flow and increases the surface temperature and reduces the heat transfer at the interface.

Fig. 8(a)-8(b) reveal that the effect of thermal conductivity variation parameter γ on the surface temperature and heat transfer against *x* with *M* = 0.01, *Pr* = 0.733, *Q* = 0.01 and *J* = 0.01. It is seen that the surface temperature increases for increasing γ. The same result is found for heat transfer from fig. 8(b). we observe from fig. 8(b) that heat transfer increases for the increasing γ.

Fig. 9(a)-9(b) deal with the effect of *Pr* on surface temperature and the rate of heat transfer with the increasing of axial distance *x* for fixed *M* = 0.01, γ = 0.01, *Q* = 0.01 and *J* = 0.01. The values of Prandtl number are proportional to the viscosity of the fluid. So the increasing *Pr* the surface temperature decreases on the plate which is shown in fig. 9(a). From fig. 9(b), we see that heat transfer increases due to increasing values of *Pr*.

Fig. 10(a), 10(b) deal with the effect of heat generation parameter *Q* on the surface temperature and heat transfer against *x* with controlling parameter *M* = 0.01, γ = 0.01, *Pr* = 0.733 and *J* = 0.01. We see that an increase value of *Q* increase fluid velocity within boundary layer which shown in fig. 5(a). So the corresponding surface temperature increase with increasing values of *Q*. The opposite result is seen for heat transfer with increasing of *Q*. Finally it is noted that the surface temperature and heat transfer increase and decrease for increasing of *Q*.

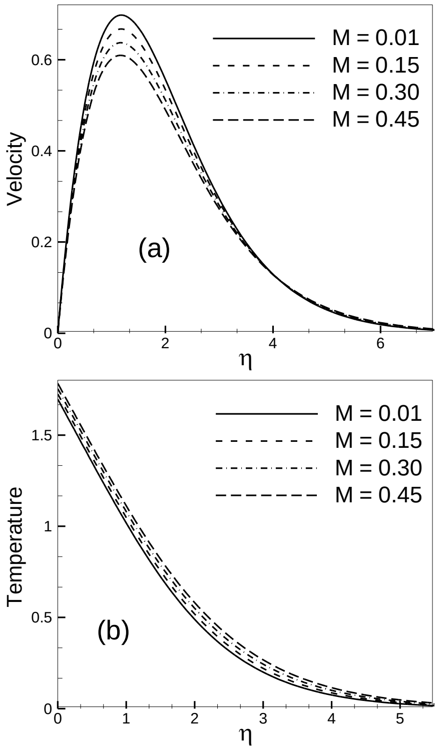

Fig 2(a). Velocity & (b) Temperature profiles against η for varying of M with γ=0.01,Pr=0.733,Q=0.01,J=0.01.

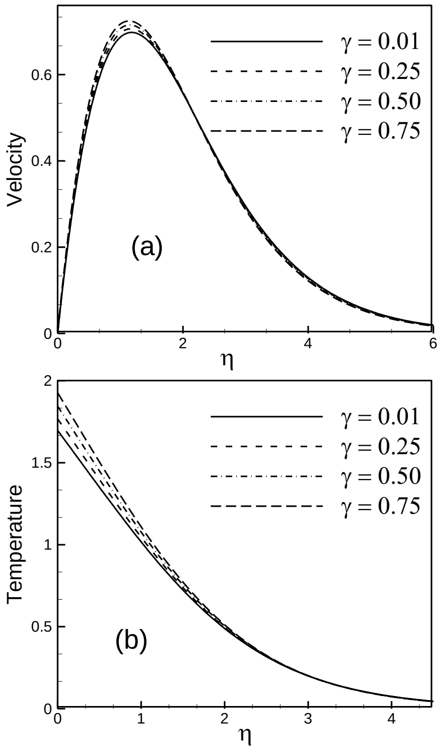

Fig 3(a). Velocity & (b) Temperature for η for varying of γ with M = 0.01, Pr = 0.733, Q = 0.01 & J = 0.01.

Fig. 11(a), 11(b) deal with the effect of joule heating parameter *J* on the surface temperature and heat transfer against *x* with *M* = 0.01, *γ* = 0.01, *Pr* = 0.733 and *Q* = 0.01. It is noted that the surface temperature and heat transfer increase and decrease for increasing values of *J*.

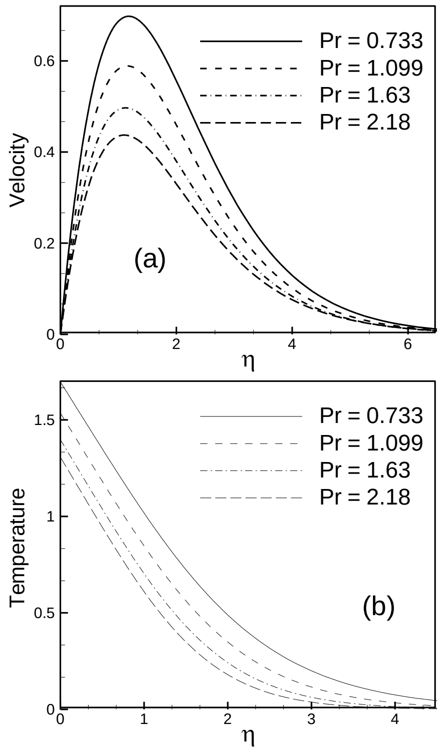

Fig 4(a). Velocity & (b) Temperature profiles against η for varying of Pr with M=0.01,γ=0.01,Q=0.01,J=0.01.

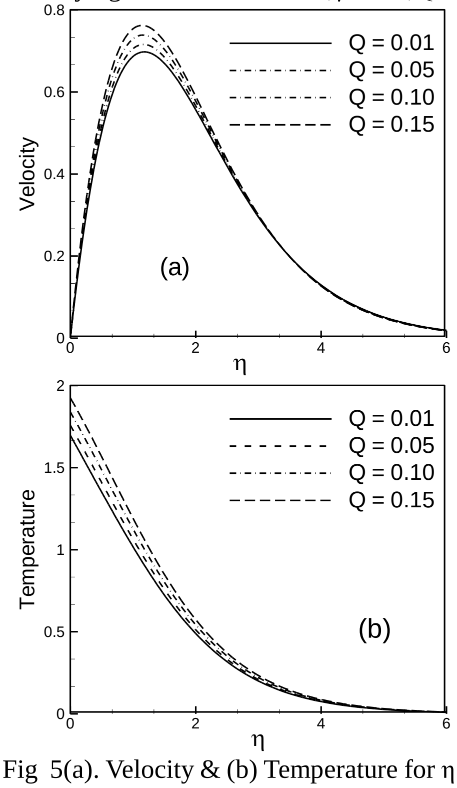

Fig 5(a). Velocity & (b) Temperature for η for varying of Q with M = 0.01, γ = 0.01, Pr = 0.733 & J = 0.01.

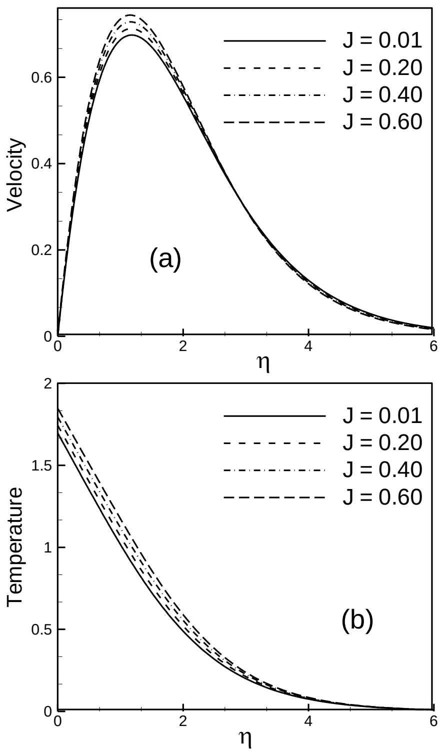

Fig 6(a). Velocity & (b) Temperature for η for varying of J with M = 0.01, γ = 0.01, Pr = 0.733 & Q = 0.01.

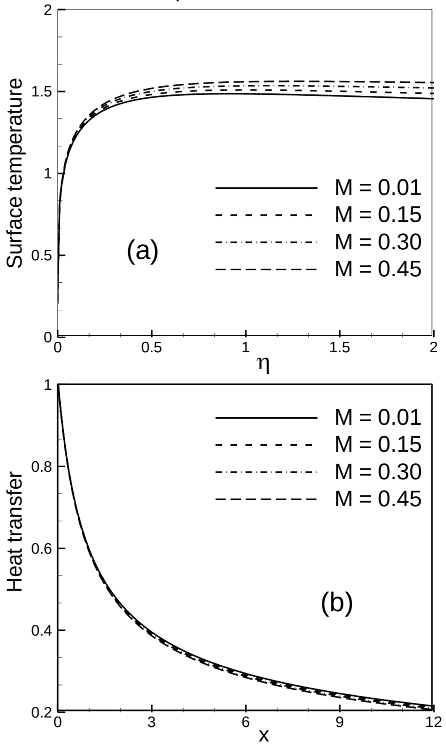

Fig 7(a). Surface temperature & (b) Heat transfer for x for varying of M with γ=0.01,Pr=0.733,Q=0.01,J=0.01.

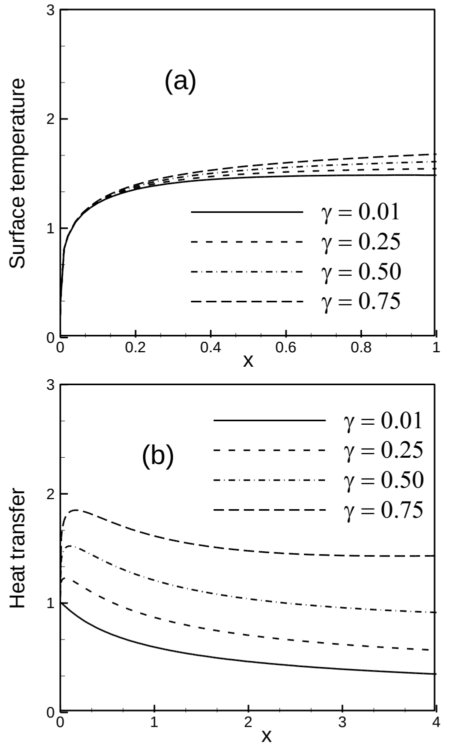

Fig 8(a). Surface temperature & (b) Heat transfer for x for varying of γ with M=0.01,Pr=0.733,Q = 0.01,J=0.01.

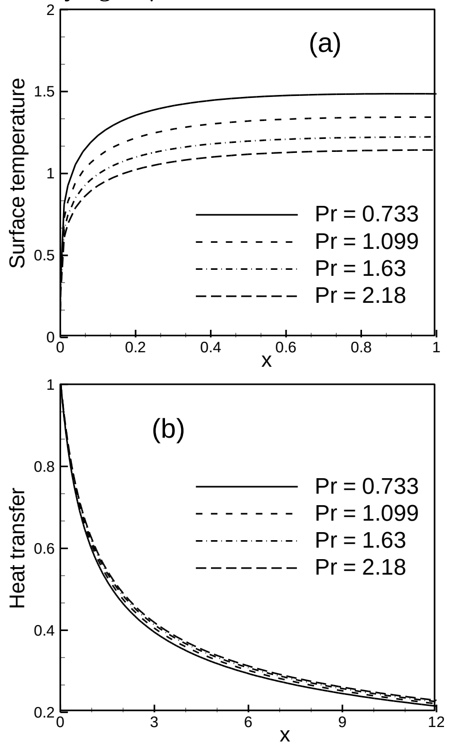

Fig 9(a). Surface temperature & (b) Heat transfer for x for varying of Pr with M=0.01,γ=0.01,Q=0.01,J=0.01.

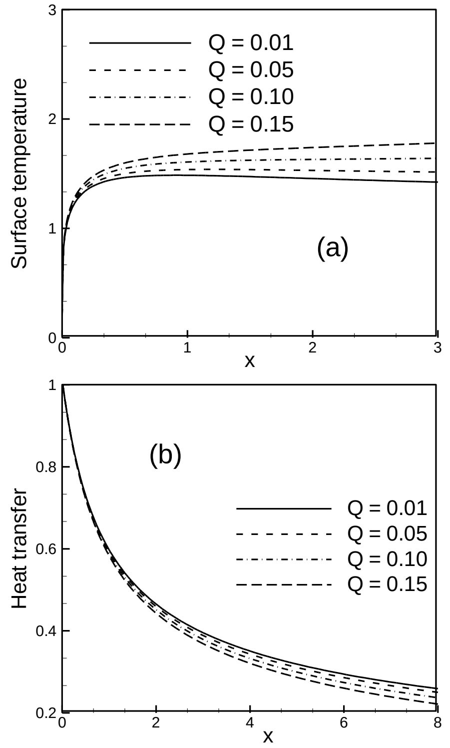

Fig 10(a). Surface temperature & (b) Heat transfer for x for varying of Q with γ=0.01,M=0.01,Pr=0.733,J=0.01.

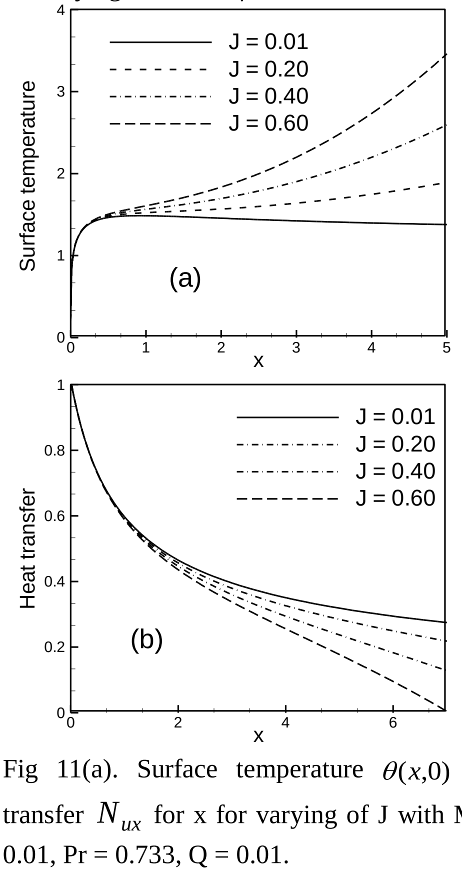

Fig 11(a). Surface temperature $\theta(x,0)$ & (b) Heat transfer N_{ux} for x for varying of J with M = 0.01, γ = 0.01, Pr = 0.733, Q = 0.01.

5. CONCLUSION

From this investigation the following conclusions may be drawn

i) The velocity within the boundary layer increases for decreasing values of *M, Pr* and for increasing values of *γ, Q* and *J*.

ii) The temperature within the boundary layer increases for increasing values of *M, γ, Q* and *J* and for decreasing values of *Pr.*

iii) The surface temperature decreases for the increasing values of *Pr* and increases for increasing values of *M, γ, Q* and *J*.

iv) Increasing values of *γ*.and *Pr* leads to an increase in the rate of heat transfer. On the other hand, this decreases for increasing values of *M, Q* and *J*.

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